

Realiza el estudio completo

$$y = \frac{x^4}{2} - 3x^2 + 1$$

1) $D = R$ No posee asíntotas ni ramas asintóticas

2) R_e $\frac{x^4}{2} - 3x^2 + 1 - y = 0$

$$x^4 - 6x^2 + 2 - 2y = 0$$

$$x^2 = t \quad t^2 - 6t + 2 - 2y = 0$$

$$t = \frac{6 \pm \sqrt{36 - 8(1-y)}}{2} = \frac{6 \pm \sqrt{28 + 8y}}{2} = \frac{6 \pm 2\sqrt{7+2y}}{2}$$

$$y + 2y > 0$$

$$y > -\frac{7}{2} = -3,5$$

$$R_e = [-3,5; \infty)$$

3) Ceros con eje $x \quad y = 0 \quad \frac{y^4}{2} - 3x^2 + 1 - y = 0$

$$x^4 - 6x^2 + 2 = 0$$

$$x^2 = t \quad t^2 - 6t + 2 = 0$$

$$t = \frac{6 \pm \sqrt{36 - 8}}{2} = \frac{6 \pm 2\sqrt{7}}{2} = 3 \pm \sqrt{7}$$

$$x = \pm \sqrt{t}$$

$$t = 3 + \sqrt{7}$$

$$x_1 = \sqrt{3 + \sqrt{7}}$$

$$t = 3 - \sqrt{7}$$

$$x_3 = \sqrt{3 - \sqrt{7}}$$

$$x_2 = -\sqrt{3 + \sqrt{7}}$$

$$x_4 = -\sqrt{3 - \sqrt{7}}$$

$$(\sqrt{3 + \sqrt{7}}, 0) \quad (-\sqrt{3 + \sqrt{7}}, 0) \quad (\sqrt{3 - \sqrt{7}}, 0) \quad (-\sqrt{3 - \sqrt{7}}, 0)$$

con eje $y \quad x = 0 \quad y = 1 \quad (0, 1)$

4) Signo

$$\begin{array}{ccccccc} + & - & + & - & + \\ \hline -\sqrt{3 + \sqrt{7}} & -\sqrt{3 - \sqrt{7}} & \sqrt{3 - \sqrt{7}} & \sqrt{3 + \sqrt{7}} & \\ \approx -2,4 & \approx -0,6 & \approx 0,6 & \approx 2,4 & \end{array}$$

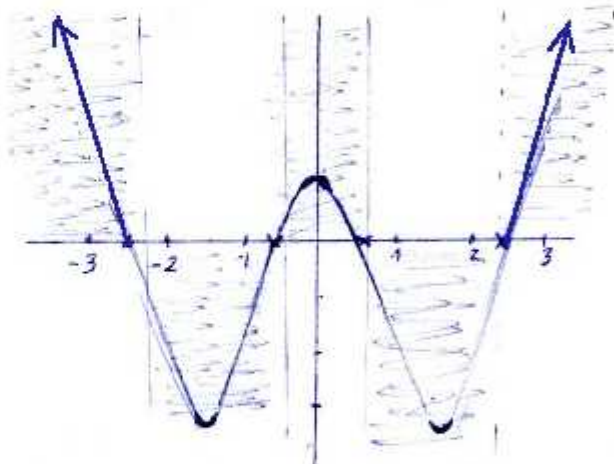
$$y(3) = 14,5 > 0$$

$$y(1) = -1,5 < 0$$

$$y(0) = 1 > 0$$

$$y(-1) = -1,5 < 0$$

$$y(-3) = 14,5 > 0$$



5) Simetría

$$f(x) = \frac{x^4}{2} - 3x^2 + 1$$

$$f(-x) = \frac{(-x)^4}{2} - 3(-x)^2 + 1 = \frac{x^4}{2} - 3x^2 + 1 = f(x)$$

Par. Simetría respecto a eje y

6) Asíntotas

verticales

$$(D = R)$$

Horizontales

$$\lim_{x \rightarrow \infty} \frac{x^4}{2} - 3x^2 + 1 = \infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^4}{2} - 3x^2 + 1 = \infty$$

No hay Hay ramas asintóticas

7) Monotonía

$$y' = \frac{4x^3}{2} - 6x = 2x^3 - 6x$$

$$2x^3 - 6x = 0 \quad 2x(x^2 - 3) = 0 \quad \left\{ \begin{array}{l} x = 0 \\ x^2 - 3 = 0 \quad x = \pm\sqrt{3} \end{array} \right.$$

Creciente $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

Decreciente $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

$$\begin{array}{ccccccc} & & & + & + & + \\ & & & 0 & & & \\ + & + & - & - & - & + & + \\ & & -\sqrt{3} & & \sqrt{3} & & \end{array}$$

$$\begin{array}{ccccccc} & & & + & + & + & \\ & & & 0 & & & \\ - & - & + & + & - & - & + & + \\ & & -\sqrt{3} & & 0 & & \sqrt{3} & \end{array}$$

8) Extremos

$$y' = 0 \rightarrow x = 0 \quad x = \pm\sqrt{3}$$

$$y(0) = 1$$

$$y(\sqrt{3}) = -3,5$$

$$y(-\sqrt{3}) = -3,5$$

$$y'' = 6x^2 - 6$$

$$y''(0) = -6 \rightarrow \text{Máximo en } (0, 1)$$

$$y''(\sqrt{3}) = 12 > 0 \rightarrow \text{Mínimo } (\sqrt{3}, -3,5)$$

$$y''(-\sqrt{3}) = 12 > 0 \rightarrow \text{Mínimo } (-\sqrt{3}, -3,5)$$

$$y'' = 0 \quad 6x^2 - 6 = 0 \quad x^2 - 1 = 0 \quad x = \pm 1 \quad y(1) = -\frac{3}{2} \quad y(-1) = -\frac{3}{2}$$

$$y'''(1) = 12x = 12 \neq 0 \quad \text{Inflexión: } (1, -\frac{3}{2}) \quad (-1, -\frac{3}{2})$$

$$y'''(-1) = 12x = -12 \neq 0$$