1/ a) $x-2=\sqrt{x+4}$

$x^2-4x+4=x+4$ Comprobación $x=0$ $-2=2 \rightarrow$ No

$x^2-5x=0$ $x=5$ $3=3 \rightarrow$ Sí

$x(x-5)=0 \rightarrow \begin{cases} x=0 \\ \boxed{x=5} \end{cases}$

b) $4^x+2^x-2=0$

$2^x=t$ $t^2+t-2=0$ $2^x=1 \longrightarrow \boxed{x=0}$

$t=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm 3}{2}=\begin{cases} 1 \\ -2 \end{cases}$ $2^x=-2 \rightarrow$ No solución

2/ a) $\lim\limits_{x\to 3}\frac{2-\sqrt{2x-2}}{x-3}=\lim\limits_{x\to 3}\frac{(2-\sqrt{2x-2})(2+\sqrt{2x-2})}{(x-3)(2+\sqrt{2x-2})}=\lim\limits_{x\to 3}\frac{4-2x+2}{(x-3)(2+\sqrt{2x-2})}=\lim\limits_{x\to 3}\frac{-2(x-3)}{(x-3)(2+\sqrt{2x-2})}=$

$=\lim\limits_{x\to 3}\frac{-2}{2+\sqrt{2x-2}}=\frac{-2}{2+2}=\boxed{-\frac{1}{2}}$

b) $\lim\limits_{x\to\infty}\left(\frac{x^2-1}{x^2}\right)^{2x}=e^{\lim\limits_{x\to\infty}\left[2x\left(\frac{x^2-1}{x^2}-1\right)\right]}=e^{\lim\limits_{x\to\infty}2x\frac{x^2-1-x^2}{x^2}}=e^{\lim\limits_{x\to\infty}\frac{-2x}{x^2}}=e^0=\boxed{1}$

3/ $C=125000$ €

5% 16 años

$M=\frac{C\frac{i}{k}\left(1+\frac{i}{k}\right)^{kt}}{\left(1+\frac{i}{k}\right)^{kt}-1}=\frac{125000\,\frac{0{,}05}{12}\left(1+\frac{0{,}05}{12}\right)^{12\cdot 16}}{\left(1+\frac{0{,}05}{12}\right)^{12\cdot 16}-1}=\boxed{947{,}10\text{ €}}$

4/ $\frac{x^2-1}{x^3-2x^2+x}+\frac{x^2+x}{x+1}-\frac{x^2+2x+1}{x^2+x}=\frac{(x+1)(x-1)}{x(x-1)^2}+\frac{x(x+1)}{x+1}-\frac{(x+1)^2}{x(x+1)}=$

$=\frac{x+1+x^3-x^2-x^2+1}{x(x-1)}=\frac{x^3-2x^2+x+2}{x(x-1)}$

5/ a) $y=\frac{x^3}{3}-\frac{3x^2}{2}+2x$ $y'=x^2-3x+2=0$ $x=\frac{3\pm\sqrt{9-8}}{2}=\frac{3\pm 1}{2}=\begin{cases} 2 \\ 1 \end{cases}$

$y''(2)=2x-3=1>0 \rightarrow$ Mínimo $y(2)=\frac{8}{3}-6+4=\frac{8}{3}-2=\frac{2}{3}$ $\left(2,\frac{2}{3}\right)$

$y''(1)=2x-3=-1<0 \rightarrow$ Máximo $y(1)=\frac{1}{3}-\frac{3}{2}+2=\frac{2-9+12}{6}=\frac{5}{6}$ $\left(1,\frac{5}{6}\right)$

6 b) $y=\frac{x^2}{x-1}$ $y'=\frac{2x(x-1)-x^2}{(x-1)^2}=\frac{2x^2-2x-x^2}{(x-1)^2}=\frac{x^2-2x}{(x-1)^2}$

$x^2-2x=0$ $x=0$, $x=2$ ++ | -- | ++ (0, 2)

$x-1=0$ $x=1$ ++++ | ++++ (1)

++ | - | - | ++ (0, 1, 2)

Creciente $(-\infty,0)\cup(2,\infty)$

Decreciente $(0,1)\cup(1,2)$

6/

x	-1	2	3
y	0	3	12

$\left.\begin{array}{l} a-b+c=0 \\ 4a+2b+c=3 \\ 9a+3b+c=12 \end{array}\right\}$ $\begin{array}{ll} E_1 & a-b+c=0 \\ E_2-4E_1 & 6b-3c=3 \\ E_3-9E_1 & 12b-8c=12 \end{array}$ $\begin{array}{ll} E_1 & a-b+c=0 \\ E_2 & 6b-3c=3 \\ E_3-2E_2 & -2c=6 \end{array}$

$c=-3$ $b=\frac{3+3c}{6}=\frac{1+c}{2}=-1$ $a=b-c=-1+3=2$ $y=2x^2-x-3$ $\boxed{y(0)=-3}$

7/ $2\,\text{sen}^2x=2+\cos x$

$\text{sen}^2x+\cos^2x=1$

$\text{sen}^2x=1-\cos^2x$

$2(1-\cos^2x)=2+\cos x$

$2\cos^2x+\cos x=0$

$\cos x(2\cos x+1)=0 \rightarrow \begin{cases} \cos x=0 \\ \cos x=-\frac{1}{2} \end{cases}$

$x_1=\text{arc}\cos 0=\begin{cases} 90^\circ\pm k\cdot 360^\circ \\ 270^\circ\pm k\cdot 360^\circ \end{cases}$

$x_2=\text{arc}\cos\left(-\frac{1}{2}\right)=\begin{cases} 120^\circ\pm k\cdot 360^\circ \\ 240^\circ\pm k\cdot 360^\circ \end{cases}$

8/

x_i	f_i	F_i	$x_i f_i$	$x_i^2 f_i$
0	7	7	0	0
1	6	13	6	6
2	8	21	16	32
3	5	26	15	45
4	2	28	8	32
5	5	33	25	125
6	6	39	36	216
7	5	44	35	245
8	5	49	40	320
9	6	55	54	486
10	8	63	80	800
	63		315	2307

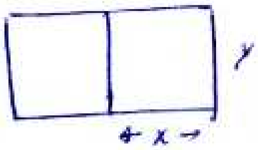
$\bar{x} = \frac{\sum x_i f_i}{N} = \frac{315}{63} = 5$

$\sigma = \sqrt{\frac{\sum x_i^2 f_i}{N} - \bar{x}^2} = \sqrt{\frac{2307}{63} - 25} = 3{,}4087$

$Q_1 = 2$ (1ª cuya F_i supera a $\frac{63}{4} = 15{,}75$)

$C_{70} = 8$ (1ª cuya F_i supera a $\frac{63}{100} \cdot 70 = 44{,}1$)

9/



$4x + 3y = 300 \qquad x = \frac{300-3y}{4}$

$S = 2xy = \frac{300-3y}{2} y = 150y - \frac{3}{2}y^2$

$S' = 150 - 3y = 0 \quad \boxed{y = \frac{150}{3} = 50\ m} \quad x = \frac{300-3\cdot 50}{4} = \frac{150}{4} = \frac{75}{2} = \boxed{37{,}5\ m}$

$S'' = -3 < 0 \rightarrow$ Máximo

10/ $x^2 - 2x > 10$

$x^2 - 2x - 10 > 0$

$x = \frac{2 \pm \sqrt{4+40}}{2} = \frac{2 \pm \sqrt{44}}{2} = \frac{2 \pm 2\sqrt{11}}{2} = 1 \pm \sqrt{11}$

++ ——— ++
$1-\sqrt{11}$ $\quad 1+\sqrt{11}$

$(-\infty, 1-\sqrt{11}) \cup (1+\sqrt{11}, \infty)$